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NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUPPLEMENTAL/SPRING EXAMINATION 2001

THIRD YEAR ELECTRONIC AND COMPUTER ENGINEERING

EE308 SIGNALS AND COMMUNICATIONS

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Duration of Examination: **TWO** hours
Instructions: Answer **THREE** questions

1.

(a)

(i) What is an LTI system?
(ii) Explain the term "orthogonal basis function" for signals.
(iii) State Parseval's theorem for periodic functions.

(b) Obtain the exponential Fourier series representation of the periodic signal $f(t) = e^{-t}$ as shown in Fig. 1. Sketch the magnitude and phase spectra of the signal.

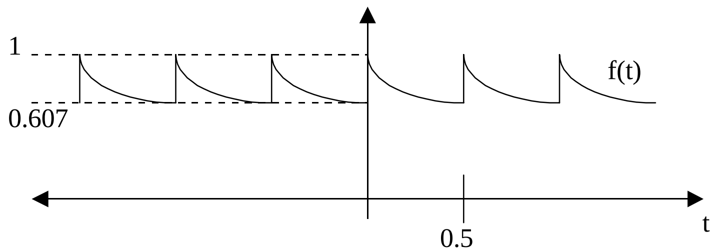

Fig. 1.

2.

(a) Obtain the trigonometric Fourier series representation of the signal $x(t) = t^2$ over the interval (0, 2), which repeats with a frequency of 0.5 Hz. What is Gibbs' phenomenon, and explain whether it applies to the signal x(t) or not.

(b) Prove that the trigonometric Fourier series of the even function s(t) as shown in Fig. 2 is given by:

$$s(t) = \frac{4}{\pi}\sum_{\substack{n=1 \\ n\,\text{odd}}}^{\infty}\frac{1}{n^2}\cos(nt) \text{ over the interval } (-\pi, \pi).$$

Define half-wave symmetry, and explain if the signal s(t) displays this property or not.

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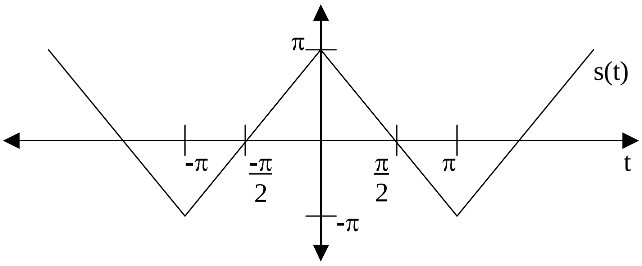

Fig. 2.

3.

(a) Name and prove the following properties of the Fourier transform:

(i) $\Im\{f^*(t)\} = F^*(-\omega)$.

(ii) If $\Im\{g(t)\} = G(\omega)$, then $\Im\{G(t)\} = 2\pi g(-\omega)$.

(iii) $\Im\{s(\alpha t)\} = \dfrac{1}{|\alpha|} S\left(\dfrac{\omega}{\alpha}\right)$.

(b)

(i) Find the Fourier transform of the function $x(t) = (1 - 2e^{-t})[u(t) - u(t-2)]$.

(ii) If $\mathrm{rect}(t) \Leftrightarrow \mathrm{Sa}(\omega/2)$, determine the Fourier transform of $\mathrm{Sa}(t/2)$.

4.

(a)

(i) Draw the magnitude and phase spectra corresponding to the frequency response of an ideal low pass filter with cutoff frequency ω_c.

(ii) What is the principle of causality? Does it hold for the impulse response of an ideal LPF?

(iii) Explain the difference between a band pass filter and the passband of a filter.

(b) Explain what is meant by the "-3 dB bandwidth" of a filter. If the magnitude of the frequency transfer function of an nth order Butterworth filter is:

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\dfrac{\omega}{\omega_0}\right)^{2n}}},$$

what is the DC gain? Compute the ratio of the –60 dB to –6 dB bandwidths for the first, second, third and fourth order Butterworth filters with ω_0 = 1 rad/s.

5.

(a)

(i) What is thermal noise? Give an approximate expression for the PSD of thermal noise.

(ii) Sketch the PSD and autocorrelation functions for both ideal and bandwidth-limited white noise.

(b) By modelling a noisy resistor as a voltage source in series with an ideal resistor, derive an expression for the RMS noise voltage out of a noisy resistor R when it is connected in parallel with a capacitor C. Make use of the fact that the power spectral density of the output noise is an even function of frequency, and also note that:

$$\int_0^\infty \frac{1}{x^2 + a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\tfrac{x}{a}\right).$$