

Figure 2: Data flow diagram of core selection process.

In this screen, the various parameters for a forward converter are entered, and using the formulae for the various factors (waveform, power, etc.), the A_p value (product of the window area, W_a , and the cross-sectional area, A_c) is calculated. When this value is obtained, the user can then click on the core type of his or her choice, and a corresponding database of core types is opened from which cores with an A_p value slightly greater than the calculated one are automatically selected. This detailed design is discussed with an example in the next section.

All core data is accessible using the sophisticated database storage and retrieval capabilities of the DataControl VBX which acts as an intermediary between the CAD package and the Microsoft Access database engine. This engine is supplied with Visual Basic and eliminates the need for the user to buy a copy of Access to run the design package.

A number of different core database files are supplied with the package, e.g. one for E-E cores, pot cores etc. If the user is not satisfied with the properties of the various cores suggested by the package, a custom core may be added with user-defined properties to suit that particular application, and this can be saved to the database file for use in future applications.

Example: Forward Converter

Reference [1] features a design example for a forward converter application with the specifications listed below.

<i>Specifications</i>	Output: 8 V, 10 A Input: 12-36 V Frequency: 25 kHz Temperature Rise, ΔT : 25 °C Efficiency, η : 90%
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Assuming equal turns in both the primary and secondary windings, the duty cycle, D , is equal to 8 V/12 V, or 0.66. The waveform factor, K , relates the RMS value of the voltage waveform, V , to the other parameters in the basic transformer equation

$$V = KfNB_m A_c \quad (1)$$

where f is the frequency of operation, N is the number of turns in a winding, B_m is the maximum operating flux density, and A_c is the cross-sectional area of the core.

K can be defined for any shape waveform. The waveform factor for a forward converter was established in [1] as

$$K = \frac{1}{\sqrt{D(1-D)}} = \frac{1}{\sqrt{0.66(1-0.66)}} = 2.12 \quad (2)$$

The power factor, k_p , for the input and output windings is given by [1]

$$k_p = \sqrt{1-D} = \sqrt{1-0.66} = 0.58 \quad (3)$$

The output power of the transformer is $P_o = (8+1)10 = 90$ W. A diode forward voltage of 1 V is assumed. This then yields a total VA rating of

$$\begin{aligned} \sum VA &= \left(\frac{1}{\eta k_{pp}} + \frac{1}{k_{ps}} \right) P_o \\ &= \left(\frac{1.72}{0.9} + 1.72 \right) 90 = 327 \text{ VA} \end{aligned} \quad (4)$$

Adding an extra 5% for the reset winding gives $\sum VA = 344$ VA.

A_p can be calculated from the formula given below [1]:

$$A_p = \left(\frac{\sum VA \times 10^4}{KB_m f K_u K_t \sqrt{\Delta T}} \right)^{1.14} \quad (5)$$

The window utilisation factor, K_u , is assumed to be 0.4, and the temperature factor, K_t , is set to 50 A/cm^{1.5}/°C^{0.5} [1], [2]. Assuming a ferrite core material, B_m is limited to 0.2 T. In this example, A_p is found to be

$$\begin{aligned} A_p &= \left(\frac{344 \times 10^4}{(2.12)(0.2)(25 \times 10^3)(0.4)(50)\sqrt{50}} \right)^{1.14} \\ &= 2.58 \text{ cm}^4 \end{aligned}$$

This corresponds to the value calculated by the design package as shown in Figure 1. The Siemens ETD 39 as suggested by the package is an E-core with an A_p value near to this and has the following specifications:

<i>ETD 39</i>	Material: Mn-Zn N27
	A_c : 1.25 cm ²
	W_a : 2.34 cm ²
	A_p : 2.93 cm ⁴

4. MINIMISING LOSSES

Figure 3 is a graphical illustration of how the different losses vary with the number of winding turns [2]. The dotted line represents the core loss, P_{fe} , the dashed line is the winding (copper) loss, P_{cu} , and the full line is the sum of these two losses, P_L . Reference [2] shows that the losses are of the form

$$P_L = P_{cu} + P_{fe} = aN^2 + \frac{b}{N^2} \quad (6)$$

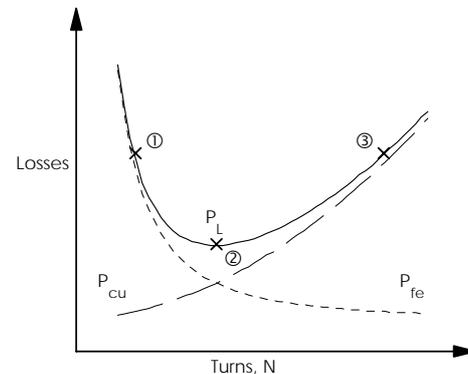


Figure 3: Relationship between losses and turns.

The partial derivative of (6) with respect to N may be taken and set to zero to yield the conditions at which the losses are minimised. In this way, the optimum number of turns is found [as shown in (7)].

$$\frac{\partial P_L}{\partial N} = 2aN - \frac{2b}{N^3} = 0 \Rightarrow aN^2 = \frac{b}{N^2}$$

$$\therefore N = \sqrt[4]{\frac{b}{a}} \quad (7)$$

As can be seen in Figure 3, the copper losses increase and the core losses decrease with increasing N since $P_{cu} \propto N^2$ and $P_{fe} \propto 1/N^2$. Also, since V and f are usually fixed, it can be seen from the basic transformer equation (1) that the maximum flux density, B_m , is inversely proportional to N .

The point marked ③ on the graph corresponds to a core operating at low frequencies. To shift a design towards the optimum point ② where the losses are at a minimum, the number of turns must be decreased with a corresponding increase in the maximum flux density. This is not feasible as B_m would then be greater than the saturation flux density, B_{sat} . Starting at point ① (core at high frequencies), optimisation may be achieved by increasing the number of turns and simultaneously reducing B_m .

The conventional formula given in (5) is based on the assumption that the total losses are equal to twice the copper losses. This is acceptable at point ③ in Figure 3, but at point ①, the total losses would be underestimated and a new formula which is based on the optimum point ② is needed. Such a formula is given in [2]:

$$A_p = \left[\frac{\sum VA}{KK_b f^{1-\alpha/2} K_u K_t \Delta T} \right]^{L.33} \quad (8)$$

where α is the coefficient of frequency in the core loss formula

$$P_{fe} = mK_c f^\alpha B_m^2 \quad \text{W/kg} \quad (9)$$

and $K_b = 1/\sqrt{135K_c}$ with K_c as given in equation (9) for m in kg. For example, for an Mn-Zn ferrite, $K_c = 1.9 \times 10^{-3}$ and $\alpha = 1.24$ giving $K_b = 1.97$ for A_p in m^4 . Normally A_p is calculated in cm^4 so $K_b = 19.7$.

5. OPTIMISING FOIL VS. ROUND CONDUCTOR

As illustrated in Figure 4 (a), a layer of insulated round wires has an equivalent foil conductor height, $h = 0.834d \times \sqrt{d/S}$ [3], where d is the diameter of the wire (excluding insulation), and S is the centre-to-centre spacing of the wires. The optimum thickness of foil may now be calculated as follows.

Consider a pulse waveform as in Figure 4 (b). This is representative of the current in a push-pull winding. I_o is related to the dc output current; for a 1:1 turns ratio, it is equal to the dc output current for a 100% duty cycle. The Fourier Series of i is

$$i(t) = I_{dc} + \hat{I}_1 \sin(\omega t) + \hat{I}_3 \sin(3\omega t) + \dots$$

$$= \frac{I_o}{2} + \frac{2I_o}{\pi} \left[\sin(\omega t) + \frac{1}{3} \sin(3\omega t) + \dots \right] \quad (10)$$

The RMS value of current is $I_{rms} = \frac{I_o}{\sqrt{2}}$.

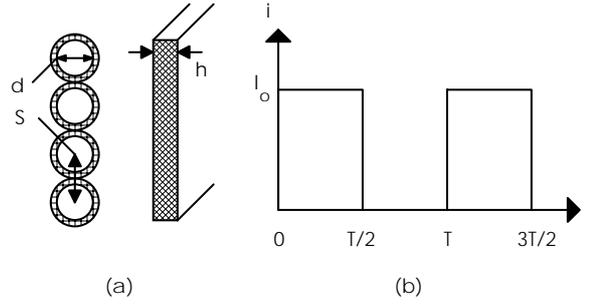


Figure 4: (a) Layer of round wires and equivalent foil height, (b) Current waveform for push-pull converter.

The total power loss is $P = R_{eff} I_{rms}^2$ which is made up of the dc component and the harmonics

$$P = R_{dc} I_{dc}^2 + R_{ac1} I_1^2 + R_{ac3} I_3^2 + \dots$$

$$= R_{dc} I_{dc}^2 + \sum_{n=1, \text{odd}} R_{acn} I_n^2 \quad (11)$$

R_{acn} is the ac resistance due to the n th harmonic, and is given by $R_{acn} = k_{pn} R_{dc}$, where k_{pn} is the proximity effect factor due to the n th harmonic [5]. Thus

$$P = R_{dc} I_{dc}^2 + R_{dc} \sum_{n=1, \text{odd}} k_{pn} I_n^2 \quad (12)$$

but $P = R_{eff} I_{rms}^2$ giving

$$\frac{R_{eff}}{R_{dc}} = \frac{I_{dc}^2 + \sum_{n=1, \text{odd}} k_{pn} I_n^2}{I_{rms}^2}$$

$$= \frac{1}{2} + \frac{4}{\pi^2} \left[k_{p1} + \frac{1}{9} k_{p3} + \dots \right] \quad (13)$$

The pulse in Figure 4 (b) is an ideal case. Normally there would be a rise time and fall time associated with the waveform so that a finite number of harmonics are required. Typically, the upper limit on the number of harmonics is

$$N = \frac{35}{t_r \%} \quad (14)$$

where t_r is the percentage rise time. For example, a 2.5% rise time would give $N = 13$.

Define R_δ as the dc resistance of a foil of thickness δ_o , where δ_o is the skin depth at the fundamental frequency of the pulsed waveform. R_{dc} is the dc resistance of a foil of thickness d and

$$\frac{R_\delta}{R_{dc}} = \frac{d}{\delta_o} = \Delta \Rightarrow \frac{R_{eff}}{R_\delta} = \frac{R_{dc}}{\Delta} \quad (15)$$

The ratio R_{eff}/R_δ is given the name k_r ; for a given waveform, R_δ is constant and k_r is the ac resistance factor for a foil of thickness d . Evidently, a plot of k_r versus Δ , the foil thickness to skin depth ratio, has the same shape as a plot of R_{eff} versus d .

The x-axis is increasing foil thickness. For $\Delta < \Delta_{\text{opt}}$, the dc resistance decreases as the thickness increases; however for $\Delta > \Delta_{\text{opt}}$, the ac effects are greater than the effect of increased thickness. The minimum ac resistance is given at Δ_{opt} and the optimum thickness is

$$d_{\text{opt}} = \Delta_{\text{opt}} \cdot \delta_o \quad (16)$$

The effective ac resistance of a foil of thickness d is

$$R_{\text{eff}} = k_r \Delta R_{\text{dc}} \quad (17)$$

From equation (12)

$$k_r = \left(\frac{0.5 + \frac{4}{\pi^2} \sum_{n=1, \text{odd}}^N \frac{k_{p_n}}{n^2}}{\Delta} \right) = \frac{0.5}{\Delta} + \frac{4}{\pi^2} \sum_{n=1, \text{odd}}^N \frac{k_{p_n}}{n^2 \Delta} \quad (18)$$

k_{p_n} is given by Dowell's formula [5]

$$k_{p_n} = \sqrt{n} \Delta \left[\frac{\text{Sinh}(2\sqrt{n}\Delta) + \text{Sin}(2\sqrt{n}\Delta)}{\text{Cosh}(2\sqrt{n}\Delta) - \text{Cos}(2\sqrt{n}\Delta)} + \frac{2(p^2 - 1) \text{Sinh}(\sqrt{n}\Delta) - \text{Sin}(\sqrt{n}\Delta)}{3 \text{Cosh}(\sqrt{n}\Delta) + \text{Cos}(\sqrt{n}\Delta)} \right] \quad (19)$$

where p is the number of layers of foil. The skin depth at the n th harmonic is

$$\delta_n = \frac{1}{\sqrt{\pi n f \mu_r \mu_o \sigma}} = \frac{\delta_o}{\sqrt{n}} \quad (20)$$

$$\Delta_n = \frac{d}{\delta_n} = \sqrt{n} \frac{d}{\delta_o} = \sqrt{n} \Delta$$

k_r is now given by

$$k_r = \frac{4}{\pi^2} \sum_{n=1, \text{odd}}^N \frac{1}{n^{\frac{3}{2}}} \left[\frac{\text{Sinh}(2\sqrt{n}\Delta) + \text{Sin}(2\sqrt{n}\Delta)}{\text{Cosh}(2\sqrt{n}\Delta) - \text{Cos}(2\sqrt{n}\Delta)} + \frac{2(p^2 - 1) \text{Sinh}(\sqrt{n}\Delta) - \text{Sin}(\sqrt{n}\Delta)}{3 \text{Cosh}(\sqrt{n}\Delta) + \text{Cos}(\sqrt{n}\Delta)} \right] + \frac{0.5}{\Delta} \quad (21)$$

k_r is plotted as a function of Δ , with p and N specified.

Regression analysis shows that the following approximation will yield the optimum value for Δ :

$$\Delta_{\text{opt}} = \frac{1}{\sqrt[4]{\left(\frac{N}{2} + 1\right) \left(\frac{p^2}{10.56}\right)}} \quad (22)$$

Example: Push-Pull Converter

$$p = 6, t_r = 2.5\%, f = 50 \text{ kHz}, N = \frac{35}{t_r \%} = \frac{35}{2.5} = 14.$$

Choose $N = 13$, since N is odd. From the graph of k_r versus Δ for $p = 6$ and $N = 13$

$$k_{r_{\text{opt}}} = 3.12 \text{ and } \Delta_{\text{opt}} = 0.43$$

$$\delta_o = \frac{66}{\sqrt{f}} = \frac{66}{\sqrt{50 \times 10^3}} = 0.295 \text{ mm}$$

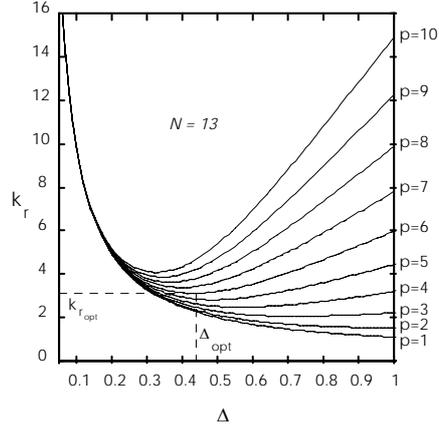


Figure 6: Resistance factor versus (foil thickness)/(skin depth) at fundamental frequency.

The optimum foil thickness is equal to

$$d_{\text{opt}} = \Delta_{\text{opt}} \delta_o = 0.43 \times 0.295 = 0.13 \text{ mm}$$

and the ac resistance is

$$R_{\text{eff}} = k_r \Delta R_{\text{dc}} = 3.12 \times 0.43 R_{\text{dc}} = 1.34 R_{\text{dc}}$$

A 0.13 mm \times 30 mm foil has the same copper area as a 2.24 mm diameter of bare copper wire, yielding $R_{\text{eff}} = 2.15 R_{\text{dc}}$.

CONCLUSIONS

This paper presents a magnetic component design package which has been developed as an application for the Microsoft Windows environment. An analysis of the transformer equations being implemented in the package is given, and the design procedure is examined. A theoretical example has been compared with an actual computerised implementation of the core selection process.

The calculation of the optimum point for minimisation of core and winding losses is illustrated, and also the choice of foil conductors as opposed to round wire for the windings is discussed. A method for obtaining the optimum thickness of a foil conductor is given along with a design example.

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